Definitions and key facts for section 5.3

We say two $n \times n$ matrices A and B are **similar** if there is an invertible matrix P such that

$$P^{-1}AP = B \text{ and } A = PBP^{-1}.$$

We call changing A to $P^{-1}AP$ a **similarity transformation.**

We say A is **diagonalizable** if A is similar to a diagonal matrix D.

Fact: An $n \times n$ matrix A is diagonalizable if and only if A has n linearly independent eigenvectors $\mathbf{v}_1, \ldots, v_n$. In this case, $A = PDP^{-1}$ with

$$P = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_n \end{bmatrix} \text{ and } D = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix}$$

where $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the corresponding eigenvalues to $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ respectively.

Note, when A is diagonalizable, it has enough eigenvectors to form a basis of $\mathbb{R}^n$. We call such a basis an **eigenbasis** of $\mathbb{R}^n$.

To diagonalize A, if possible we

1. Find all eigenvalues of A.
2. Find as many linearly independent eigenvectors as possible for each eigenvalue.
3. If there are less than n such eigenvectors, A is not diagonalizable. Otherwise,
4. place the n linearly independent eigenvectors into a matrix P and place the corresponding eigenvalues (repeating as necessary) into the diagonal entries of a matrix D.
5. Conclude A is diagonalizable with $A = PDP^{-1}$.

Fact: If A has n distinct eigenvalues then A is diagonalizable.